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Giant Resonances in Normal and Exotic Nuclei

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Abstract. Using a Time-Dependent Hartree-Fock approach, the response functions of nuclear giant resonances are calculated. We examine isovector and isoscalar giant monopole resonances in calcium isotopes ranging from the very neutron-deficient to neutron-rich isotopes. An analysis is made of the link between the neutron-proton radius difference in the isotopes and the energy of the different resonance modes. A strong correlation is found in the case of the isoscalar monopole, and a weaker correlation in the isovector mode. The analysis of the isovector mode is complicated by a strong coupling with the isoscalar vibrations.

Keywords. giant resonances, time-dependent hartree-fock, neutron skin

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1. Introduction

Giant resonances in nuclei are small amplitude excitation modes which occur at order 10 MeV above particle emission threshold. A rich variety of giant resonances have been observed and classified according to their quantum numbers [1]. Their interest rests in many reasons, amongst which their ability to probe underlying structure [2], and as a test of theoretical models. They are also a probe of nuclear matter properties. In particular, the

Isoscalar Giant Monopole Resonances (ISGMR) can be related to the incompressibility modulus of nuclear matter [3].

2. Time-Dependent Hartree-Fock

The usual microscopic approach to giant resonances is to begin from a static Hartree-Fock mean field state, and build the appropriate excitations upon it. The basic approach from this point of view is the Random Phase Approximation (RPA) [4], and its variants [5]. RPA is usually derived as the small amplitude limit of the time-dependent Hartree-Fock approximation [6], and which method one applies is a matter of choice and computational complexity. In spherically symmetric systems, and so one dimension, RPA is efficient. However, in deformed systems TDHF is the method of choice, and it is method used in this paper. The time-dependent Hartree-Fock equations [6,7]

$$i\hbar\dot{\rho} = [\hat{h}, \rho] \tag{1}$$

are obtained from a variational principle. In these equations $\hat{h}$ is the Hartree-Fock Hamiltonian, which we always take to be a Skyrme mean field [8]. We solve them by finding a static Hartree-Fock solution, evolving the single particle wavefunctions in time with an external perturbation to stimulate the giant resonance of interest. The external perturbation is of the form

$$W_{\text{ext}} = \hat{F} f(t) \tag{2}$$

where F is the functional form of the resonance operator, e.g. r^2 for isoscalar monopole resonance, $rY_{10}\tau_z$ for isovector dipole resonance etc. The time profile $f(t)$ is take to be a Gaussian of short duration, which allows a range of energies to be excited in one TDHF calculation, covering the energy scale of interest. The same spatial function is then used to measure, as a function of time, the response of the nucleus to the external perturbation. The Fourier transform of this time-dependent function can be used to analyse the response in terms of frequency;

$$F(t) = \text{tr}\{\hat{\rho}\hat{F}\} \quad \longrightarrow \tilde{F}(\omega) = \int dt e^{\imath\omega t} F(t) \tag{3}$$

Further details of the time-evolution procedure can be found in [9].

A brief word is in order for the boundary conditions used in the calculation; Each single particle TDHF state is represented on a spatial grid of a necessarily finite extent. Giant resonances exist in the continuum, and decay by particle emission. As one iterates in time, a significant amount of a wavefunction's probability can reach the edge of the computational box. At such a point, it should continue out as if the artificial box edge were not there. This is achieved in the present calculations by a combination of a large box, with an optimized complex absorbing potential far away from the nucleus. This approach is similar to those commonly applied in nuclear and atomic physics [10,11].

3. Linear Response and Strength Functions

The observable of interest is the strength function. In the case of the (isovector) giant dipole resonance, this corresponds to the well-known photoabsorption strength. The strength function is defined in general as

$$S(E) = \sum_{\nu} |\langle\nu|F|0\rangle|^2 \, \delta(E - E_\nu). \tag{4}$$

where the states $|\nu\rangle$ denote the eigenmodes of the nuclear oscillations. The strength function can be written in the spectral representation by using the Fourier form of the delta function

$$S(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \sum_{\nu} |\langle\nu|F|0\rangle|^2 \, e^{i(E-E_\nu)t}. \tag{5}$$

This integral can be performed separately for positive and negative t by inclusion of appropriate infinitesimal convergence factors:

$$S(E) = \frac{1}{2\pi} \sum_{\nu} |\langle\nu|F|0\rangle|^2 \left(\frac{1}{E - E_\nu + i\delta} - \frac{1}{E - E_\nu + i\delta} \right), \tag{6}$$

The bracketed term resulting from the integration is a c-number which can be placed inside the matrix element, and the E_ν become associated with their eigenkets, and can be replaced by the Hamiltonian

$$S(E) = \frac{1}{2\pi} \langle 0|F \left(\frac{1}{H - E + i\delta} - \frac{1}{H - E - i\delta} \right) F|0\rangle. \tag{7}$$

Dirac's formula states [12]

$$\frac{1}{x - x' \mp i\epsilon} = \mathcal{P}\frac{1}{x - x'} \pm i\pi\delta(x - x') \tag{8}$$

so that in (7) the real parts cancel and the result is twice the imaginary part of either term;

$$S(E) = -\frac{1}{\pi} Im \langle 0|F \frac{1}{H - E + i\delta} F|0\rangle. \tag{9}$$

This representation of the strength function makes it clear that a small imaginary term has been added to the Hamiltonian in order to ensure the correct boundary conditions, justifying the use of an imaginary potential to overcome artificially-imposed hard walls in the TDHF calculation.

Using linear response theory [13], it can be shown that the strength function is related to the Fourier transform of the fluctuation of the expectation value of the monopole operator divided by the Fourier transform of the time profile of the external perturbation;

$$S(\omega) = -\frac{1}{\pi} Im \int \mathrm{d}^3\mathrm{r} \frac{\delta\langle \mathrm{F}(\mathrm{r}, \omega)\rangle}{\mathrm{f}(\omega)} \tag{10}$$

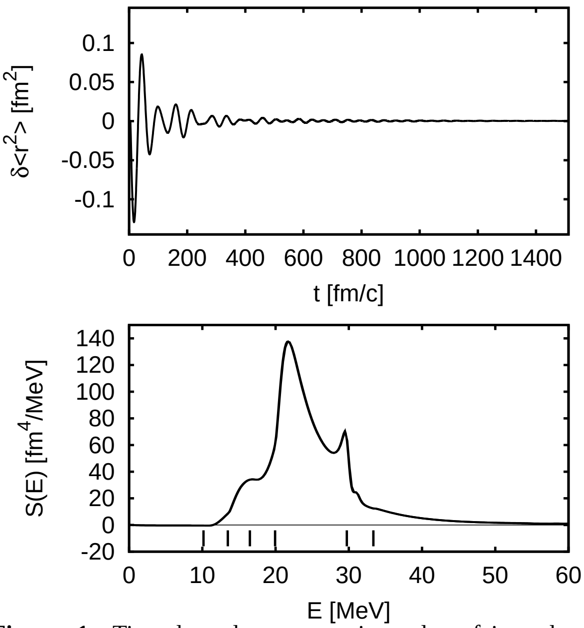

Figure 1. Time-dependent expectation value of isoscalar monopole operator (top panel) in ^{16}O, following an ISGM kick, and the associated strength function (lower panel). The single particle energies from the underlying static Hartree-Fock calculation are shown below the strength function. The Skyrme force SkM* was used.

Note that the spatial form of the function $F(r,t)$ is the just the same as the operator which caused the excitation in the TDHF code, and whose expectation value is being observed. This representation of the strength function is directly calculable from the TDHF output. An example is shown in figure 1, which shows, in the upper panel, the time evolution of the function $F(r,t) = \langle r^2(r)\rangle$ for the isoscalar giant monopole resonance, following an isoscalar monopole kick in ^{16}O. The damped nature of the resonance is clearly seen in the upper panel. The lower panel of figure 1 shows the strength function (10). Also shown, at the bottom of the lower panel, are the single particle energies from the underlying static Hartree-Fock calculation. While the strength function arises from a complicated coherent superposition of many 1p-1h states on top of the Hartree-Fock ground state depending on the residual interaction, one can see the approximate connection between the single particle energies and the appearance of structures in the strength function. This occurs since one needs to supply at least the single-particle energy of a given level to see an (unbound) resonance state.

4. Isospin structure of Giant Monopole Resonances

The strength functions for isoscalar and isovector giant monopole resonances in a series of Calcium isotopes is shown in Figure 2.

The chain of isotopes stretches from the proton-rich ^{34}Ca through stable isotopes to the

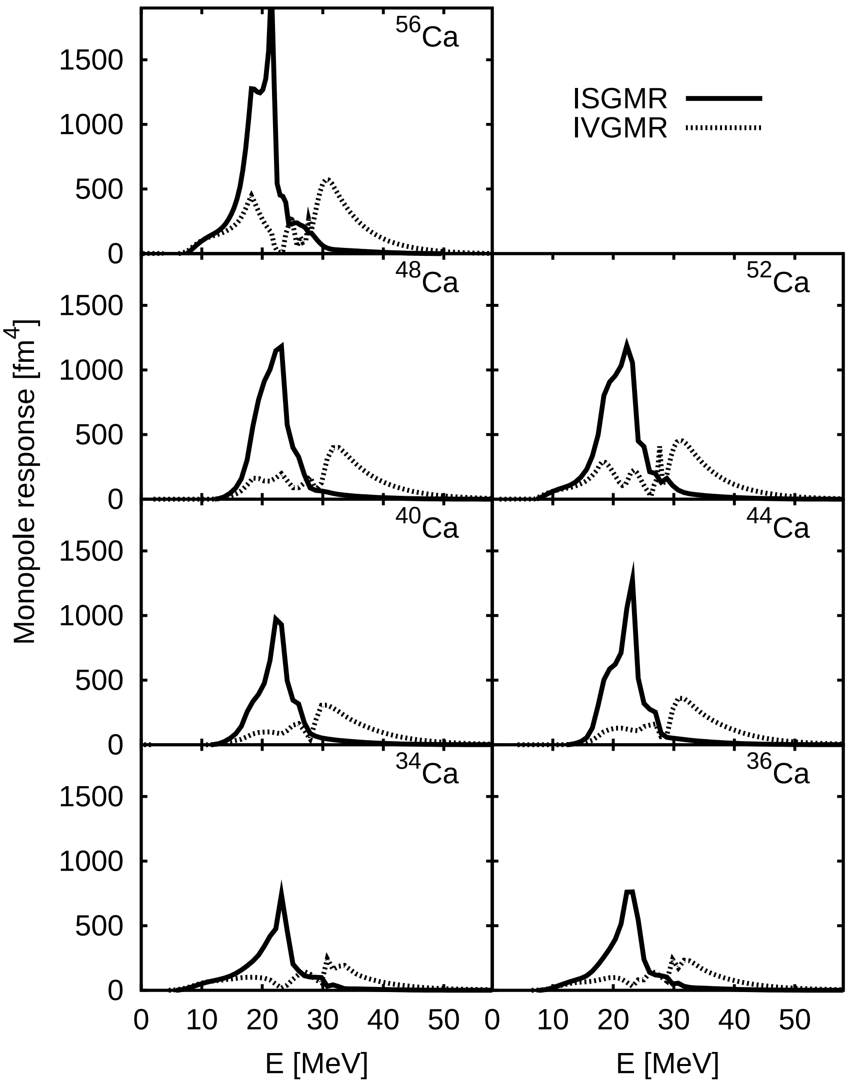

Figure 2. Isoscalar (ISGMR) and Isovector (IVGMR) Giant Monopole Resonance strength functions.

	^{34}Ca	^{36}Ca	^{40}Ca	^{44}Ca	^{48}Ca	^{54}Ca
$\overline{\mathrm{E}}_{ISGMR}$	21.3	22.0	22.8	22.7	22.4	21.1
$\overline{\mathrm{E}}_{IVGMR}$	25.9	28.5	30.7	31.1	30.7	26.6
$\|r_p - r_n\|$	0.193	0.106	0.049	0.069	0.155	0.334

Table 1. Energies in MeV of isoscalar and isovector resonances in Calcium isotopes using energy-weighted moment equation (12). Also shown is the absolute difference between the proton and neutron rms radii, in units of fm.

very neutron rich ^{56}Ca. One can see in the least stable isotopes that the strength function stretches down to low energy, indicating that the nucleons nearest the fermi surface are very weakly bound. In the case of the isovector giant monopole resonances, one sees a large strength building up at the isoscalar monopole resonance energy as one examines very neutron-rich isotopes. This is due to the fact that isospin is not a good quantum number in such nuclei, and the ISGMR and IVGMR are not the normal modes of oscillation, but are strongly coupled. Applying an isovector perturbation will excite a combination of isoscalar and isovector motion, and vice-versa. Note that these TDHF calculations of the IVGMR in Calcium isotopes agree with the RPA treatment of Hamamoto et al. [14].

Properties of giant resonances are often related to the energy-weighted moments of the strength functions, defined by

$$m_n = \int E^n S(E)\, dE, \tag{11}$$

from which a common way of characterizing the energy of a giant resonance is in the form

$$\overline{\mathrm{E}} = \sqrt{m_1/m_{-1}}. \tag{12}$$

The mean energies for the presented calcium isotopes are calculated in this way, and presented in Table 1. Since giant resonances are in general known to be sensitive to the relative radii of neutrons and protons in a nucleus [15,16], also presented in Table 1 is the absolute difference in rms neutron and proton radius (the skin thickness) in each isotope under consideration.

The skin thickness is plotted against the values of $\overline{\mathrm{E}}$ for both isoscalar (left panel) and isovector (right panel) in Figure 3. The strong correlation between the skin thickness and the isoscalar resonance energy is obvious. A straight-line fit is poorer in the case of isovector giant monopole resonances, as seen in the right panel, but the trend is still obvious that increasing the neutron or proton skin thickness reduces the resonance energy.

A possibly fruitful future analysis of the isospin nature of these giant monopole resonances would be to find the true normal modes of oscillation, which will be some combination of isoscalar and isovector modes. In the present work, the analysis of particularly the isovector oscillations is complicated by the seemingly large coupling to the isoscalar mode.

5. Conclusions

The methods of using Time-Dependent Hartree-Fock to calculate strength functions for nuclear giant resonances has been described, and the particular application to isovector

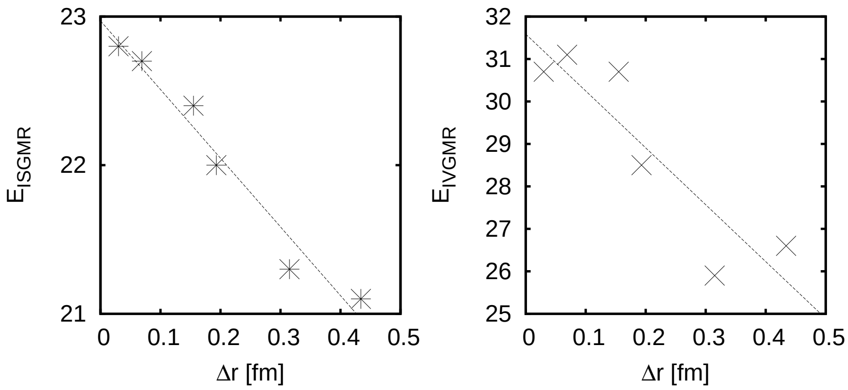

Figure 3. Correlation between difference in proton and neutron radii, Δr, and the energies of isoscalar and isovector giant monopole resonances in calcium isotopes. Energies are calculated according to (12)

and isoscalar monopole resonances in calcium isotopes demonstrated. Coupling between isovector and isoscalar modes is apparent, and a determination of the normal modes of oscillation in terms of isospin component should be made for these nuclei.

6. Acknowledgments

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