# **Nonlinear Nuclear Response Through Time-Dependent Hartree-Fock**

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**Abstract.** Time-dependent mean field methods can give a microscopic (at the level of nucleon wave functions) description of nuclear dynamics over short time-scales. Such dynamics can include compound nuclear states in which each nucleon carries a significant fraction of the internal motion. In this work, we concentrate on the example of nuclear response to the dipole field, as suitable for e.g. level density studies, looking in the present case to large-amplitude non-linear excitations that could lead to photo-fission. We give sample (nonfissioning) calculations in Th-232 motivated by recent experimental work, and suggest next steps for a more realistic approach.

### **1 Introduction**

Time-dependent mean-field methods give a microscopic picture of atomic nuclei at a level of approximation suitable for many, but not all, situations of interest [\[1\]](#page-2-0). Low energy phenomena are generally accessible to a mean-field picture, where the Pauli exclusion principle contributes to a nucleon mean-free-path that can exceed the nuclear size. Then, the collision of nucleons with the walls of the mean-field potential can account for many of the most important dynamical effects, including during the creation and evolution of the compound nucleus. Several codes have been produced over the years, which implement some version of the time-dependent mean-field equations  $[2-6]$  $[2-6]$ . In the present contribution, we use the author's Sky3D v1.2 [\[5\]](#page-2-3) to look at nonlinear response of nuclei to an external field.

### **2 Nonlinear Response**

One application of TDHF-like theories is in the linear or nonlinear response of a nucleus to an external perturbation. A most typical example in nuclear data where TDHF may be used in in the gamma-ray strength function [\[7\]](#page-2-4), though here a linear response usually suffices and e.g. RPA-level theories may be used. Examples of recent experimental observations demanding nonlinearity include large amplitude response to dipole photons resulting in photofission of U-238 and Th-232 [\[8\]](#page-2-5). We have conducted exploratory response calcualtions of the case of Th-232 at the level of time-dependent Hartree-Fock with the frozen pairing approximation, using Skyrme force SkM\* with Volume Delta Interaction pairing [\[5\]](#page-2-3). Figure [1](#page-1-0) shows the

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<span id="page-1-0"></span>Figure 1. Response of Th-232 to an isovector dipole boost of various magnitudes. for boost parameters  $k = 0.001$  to  $k = 0.1$  the isoscalar dipole response is linear, or close to linear, as seen by the overlapping lines. for  $k = 0.5$  significant non-linearity is seen. The linear isovector response is shown for reference.

isoscalar (IS) dipole response to the isovector (IV) dipole field. Here, the standard higherorder isoscalar dipole response [\[9\]](#page-2-6) is followed as a function of time

$$
D_{IS}(t) = \sqrt{3} \left( r^3 - \frac{5 \langle r^2 \rangle}{3} r \right) Y_{10}.
$$
 (1)

The boost used is a standard isovector operator *DIV* [\[5\]](#page-2-3) applied to each wave function with operator  $\exp ikD<sub>IV</sub>$  where *k* is the boost strength. We use an IV boost since this is the leading field in real photons, and follow the IS response, since fissioning products generally match the isospin properties of parent nuclei in which protons and neutrons overlap as much as possible as understood through the sign of the nuclear symmetry energy.

The response shows that for the low-boost regime, an induced isoscalar response to the isovector boost is linear in the sense that a boost of ten times the strength is results in a response of ten times the amplitude, noting that the  $y$ -axis quantity features division by the amplitude *k* so complete linearity of resposne would mean fully overlapping response lines. Note that the short-time IS response grows quadratically as expected in such a mode coupling, while the IV response is linear in time at short times.

#### **2.1 Fission**

At the largest amplitude, fission should occur (as experimentally observed). For this, one should go beyond the TDHF+frozen pairing approximation. While "boost induced fission" has been observed in such TDHF+frozen pairing codes [\[10\]](#page-2-7), unrealistically large boosts are required to overcome the collective paths unavaiable through the fixed occupation numbers. A more realistic calculation would include some physical method to allow dynamically changing occupations [\[11\]](#page-2-8). We plan a future use of the LISE code [\[6\]](#page-2-2) to explore photofission.

## **3 Conclusion**

We have given some exploratory results in an area - large amplitude response to dipole radiation ultimately leading to fission - where time-depdendent mean field might provide a reasonable description. The crossover region from linear to nonlinear reponse has been explored in time-dependent Hartree-Fock with the frozen pairing approximation. A future study with genuine photofission will adopt a more complete theory.

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